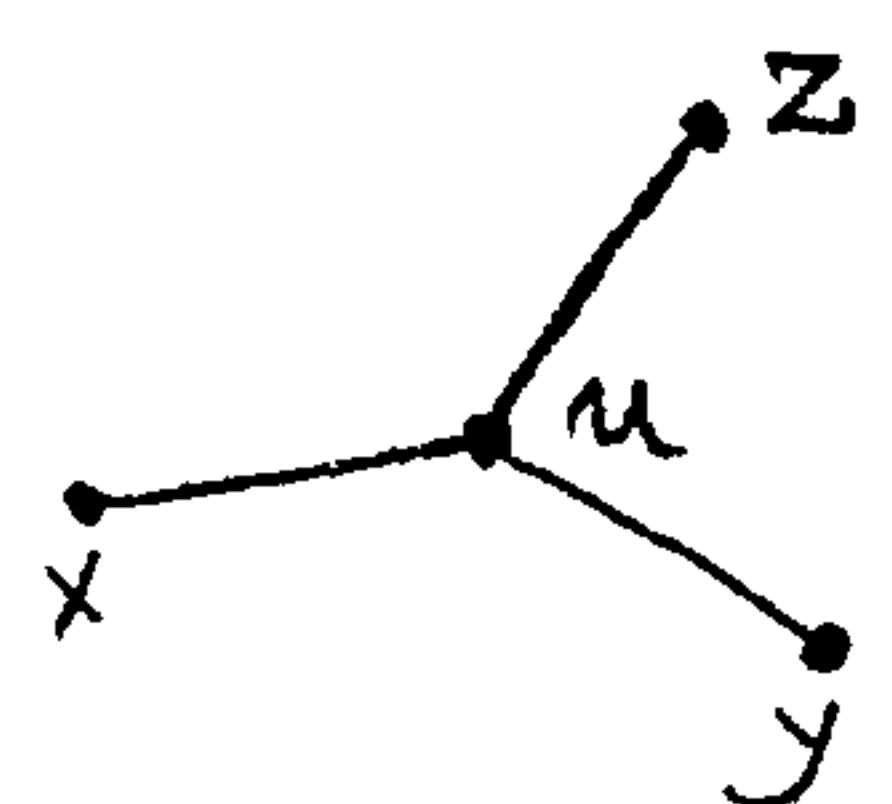


B. Momentum space Feynman integrals

The propagation of a free scalar particle is given by (5.6). Local interactions of a single scalar field are characterized by 3-vertex, 4-vertex, ... coupling strengths

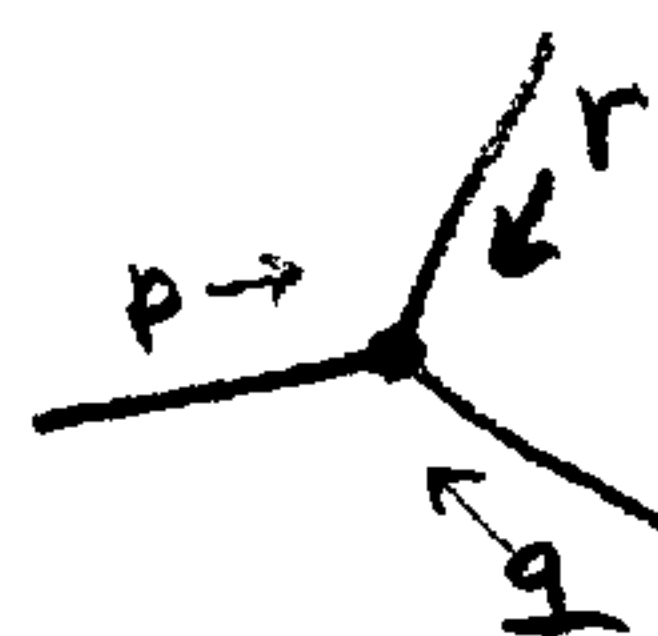


$$= g \int \frac{d^d p d^d q d^d r}{(2\pi)^{3d}} \frac{e^{ip \cdot x}}{p^2 + m^2} \frac{e^{iq \cdot y}}{q^2 + m^2} \frac{e^{ir \cdot z}}{r^2 + m^2} \int d^d x e^{-iu \cdot (p+q+r)}$$

The structure of a typical Feynman integral is already apparent. Each external leg carries a phase factor $e^{ip \cdot x}$. We can get rid of those by going to the momentum space

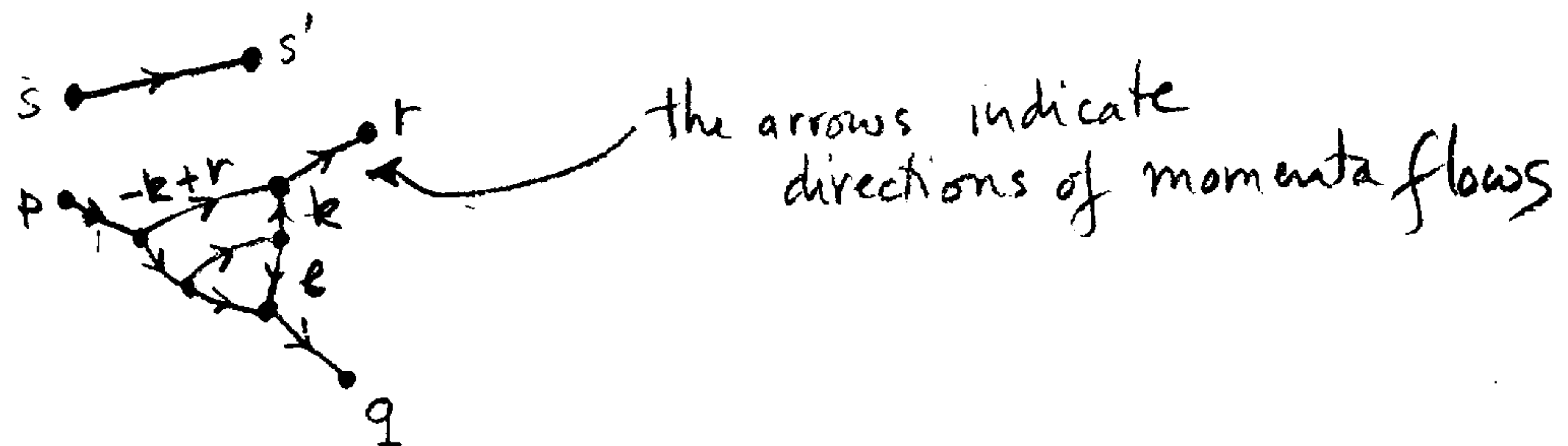
$$G(p, q, r, \dots) = \int d^d x d^d y d^d z \dots e^{-i(p \cdot x + q \cdot y + r \cdot z + \dots)} G(x, y, z, \dots)$$

Integration over the position of each vertex yields a momentum conservation δ -function



$$= (2\pi)^d \delta(p+q+r), \dots$$

Most of the vertex δ -functions can be integrated; what remains is an overall momentum conservation δ -function per each connected component, and a loop-momentum integration per each independent loop:



$$= (2\pi)^d \delta(s-s') \frac{1}{s^2+m^2} (2\pi)^d \delta(p-q-r) \frac{1}{p^2+m^2} \frac{1}{q^2+m^2} \frac{1}{r^2+m^2}$$

$$\int \frac{d^d k}{(2\pi)^d} \frac{d^d e}{(2\pi)^d} \frac{1}{(q+k)^2+m^2} \frac{1}{(k+e)^2+m^2} \frac{1}{k^2+m^2} \frac{1}{(r-k)^2+m^2} \frac{1}{(q-e)^2+m^2} \frac{1}{e^2+m^2}$$

Feynman integrals for particles with spins, different masses, color, different couplings, etc., differ only by extra numerator factors.

Though easiest to write down, the momentum space integrals are not ^{the} most economical form of Feynman integrals for evaluations purposes.

C. PARAMETRIC REPRESENTATION

It is intuitively clear that the value of a Feynman diagram cannot depend on ~~the choice of the origin of an internal loop momentum integration~~ the choice of the origin of an internal loop momentum integration. The internal momenta are integrated over their entire range, and that is that. In a momentum space Feynman integral there are many redundant variables which correspond to its invariance under momentum shifts, or momentum re-routings.

Removal of these redundant integration variables is attained by parametric representation.

Feynman propagator for a scalar particle, $1/(p^2+m^2)$, is a factor that suppresses large excursions in the momentum space. It does not care about the individual momentum components, it depends only on the magnitude of p^2 . The idea of the parametric representation is to replace the momentum variables by parameters z_i :

$$\frac{1}{k^2+m^2} = \int_0^\infty dz e^{-z(m^2+k^2)}$$

This has the virtue of turning momentum integrals into gaussian integrals, the only integrals that we know how to do. The result is of form

$$\underbrace{\int \frac{d^d r_1}{(2\pi)^d} \frac{d^d r_2}{(2\pi)^d}}_{\text{loops}} \cdot \frac{\text{lines}}{\prod} \frac{1}{k_\lambda^2 + m^2} = \int \frac{dz G}{\mathcal{V}^{d/2}} e^{-\sum z_i m_i^2 - G(q_i, q_j)}$$

\swarrow effective mass
 \swarrow effective momentum squared
 generalized "propagator" for the entire diagram

For large momenta, the dominant contribution is from $z \rightarrow 0$; the ultraviolet region of the momentum integral is transferred to ^{the} small z region of the parametric integral.

This behavior suggests an interpretation of Feynman diagrams which is very useful in developing intuition: electric circuit analogy. We think of k_i as an electric current, and z_i as the resistance of i -th line. It is easy to check that z 's add up like resistances. If the current flows through a series of lines

$$\begin{array}{c} 1 \quad 2 \quad 3 \\ \bullet \text{---} \bullet \text{---} \bullet \text{---} \bullet \end{array} \propto e^{-z_1 k^2 - z_2 k^2 - z_3 k^2} = e^{-z_{123} k^2}$$

the resistances z_i add. If the current flows through a parallel network

$$\begin{array}{c} 1 \\ \circ \\ 2 \\ \circ \\ \vdots \\ n \\ \circ \end{array} \begin{array}{c} \leftarrow q \\ \rightarrow q' \end{array} = \int dx dy \int \prod \frac{dk_i}{(2\pi)^d} e^{-\sum_i (z_i k_i^2 + i(x-y) \cdot k_i)} e^{-ix \cdot q + iy \cdot q'}$$

we evaluate by going first to the configuration space

$$\begin{aligned} &= \dots \int \prod \frac{dk_i}{(2\pi)^d} e^{-\sum_i z_i \left[\left(k_i - \frac{i(x-y)^2}{4z_i} \right)^2 - \frac{(x-y)^2}{4z_i} \right]} \dots \\ &= \dots \frac{1}{\pi (4\pi z_i)^{d/2}} e^{-\frac{(x-y)^2}{4} \sum_i \frac{1}{z_i}} \dots \end{aligned}$$

Going back to the momentum space we conclude that for a parallel network the resistances add as

$$\frac{1}{G} = \sum_i \frac{1}{Z_i} ,$$

just as in an ordinary electric circuit.

The parametric representation of Feynman integral has many virtues. It is the most economical representation; the functions \mathcal{U} and G depend only on the topology of the diagram, and all arbitrariness due to $SO(d)$ invariances and freedom of momenta routings is eliminated. The divergences are neatly separated; ultraviolet divergences arise from \mathcal{U} , and infrared divergences from the exponent.

D. Configuration space Feynman integrals

For massless particles the propagator takes a very simple form (exercise 5.A.3):

$$\Delta(x, y) = \frac{\Gamma(\frac{d}{2}-1)}{4\pi^{d/2}} \frac{1}{|x-y|^{d-2}}$$

The simplicity of the configuration space massless propagators is due to the conformal invariance of the theory,