Dynamics of the spin of atomic ensemble in the laser wave

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April 29, 2004

Abstract

The possibility to realize kicked top using ensemble of atomic spins is considered. The average values of spin observable were calculated.

1 Introduction

Lets consider experiment shown on the Fig. 1. The sample of $N \sim 10^7 - 10^8$ atoms is placed in the weak magnetic field directed along z axis and illuminated by the short pulses of linearly polarized light with the period τ. Part of the of light is reflected back to the sample of atoms. $\lambda/8$ wave plate with the axis aligned along the initial polarization of light is inserted into the beam just before the mirror to convert the rotation of linear polarization in the input beam to the admixture of circular polarization in the output beam. Another part of the light beam is sent to the polarimeter to monitor the angle of polarization plane rotation.

The refraction index in the sample of spin 1/2 atoms for the light traveling in z directions is equal to

$$n_{\pm} = 1 + \frac{\rho\alpha(\Delta)}{2\epsilon_0}\left(\frac{2}{3} \pm \frac{F_z}{3F}\right)$$

where $\pm$ corresponds to the left and right circular polarization, ρ is the density of atoms, $\alpha(\Delta)$ is the polarizability of atoms for the light detuned from the resonance by Δ, F is the total spin of atoms summed over all atoms in this ensemble, F_z is the z component of the spin.

The angle of polarization plane rotation is equal to

$$\phi = \frac{1}{2}(n_+ - n_-)kl = \frac{\rho\alpha(\Delta)}{6\epsilon_0}\frac{F_z}{F}kl$$

where k is the wave vector of light, l is the length of the sample. For large detuning we can write

$$\phi = \frac{\kappa\Gamma F_z}{6\Delta F}$$

where κ is the optical depth of the sample on resonance, Γ is the natural width of the atomic line.

If the polarization of light is not parallel to the axis of the wave plate then light gets admixture of circular polarization proportional to the angle of polarization plane rotation.

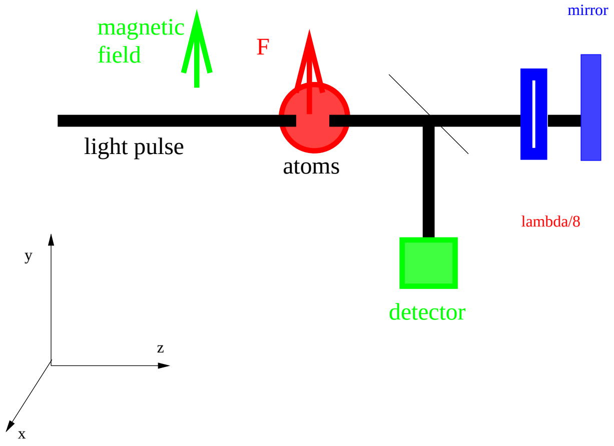

Figure 1: Schematic of the system

The effective energy of atom in the left (right) circularly polarized light wave is equal to

$$U_{eff} = \alpha(\Delta)E^2(\frac{2}{3} \pm \frac{f_z}{3f})$$

where E is the electric field strength, $f = 1/2$ is the spin of the single atom. Due to the terms proportional to f_z atomic spin precess in the laser beam about the direction of the light propagation.

Combining interaction of atomic ensemble with the light pulses and constant magnetic field we can write the Hamiltonian that describes the evolution of collective atomic spin as follows.

$$\hat{H} = \frac{p'}{\tau}\hat{F}_y + \sum_n \beta' \frac{\hat{F}_z^2}{F}\delta(t - n\tau) \tag{1}$$

where $p'/\tau = g\mu B$ and $\beta' = \frac{2\hbar}{9\pi f}\frac{N_p}{N_a}(\frac{\kappa\Gamma}{\Delta})^2$. Here N_p is the number of photons in each pulse, N_{at} is the number of atoms in the sample.

If we measure the time in the units of τ and energy in the units of $\hbar/\tau$ then the dimensionless Hamiltonian takes the form

$$\hat{H} = p\hat{F}_y + \sum_n \beta \frac{\hat{F}_z^2}{2F}\delta(t - n) \tag{2}$$

here $p = g\mu B\tau/\hbar$, and $\beta' = \frac{4}{9f\pi}\frac{N_p}{N_a}(\frac{\kappa\Gamma}{\Delta})^2$.

This is the the Hamiltonian of the kicked top. Parameter p here determines the angle of spin rotation in the magnetic field between the kicks, β gives the strength of the kick. The goal of this project is to study this system for the experimentally feasible parameters and find the expectation values of $\langle F_z^2 \rangle$ and $\langle F_y^2 \rangle$.

2 Estimates

If optical density $\kappa = 50$, natural linewidth $\Gamma = 5Mhz$, detuning $\Delta = 500Mhz$, density of atoms in the dark spot MOT [2] is $\rho = 10^{12}cm$, $l = 1cm$, $k \sim 10^7 cm^{-1}$ and $A = 10^{-4}cm^2$, then the spin rotation in the magnetic field $B = 1G$ for the time $t = 10^{-7}s$ ($\sim \pi/2$) is comparable with the change of spin in the light pulse with the pulse energy about $W \sim 10^{-10}J$. We assume that the number of atoms in the atomic sample is $N_a = 10^8$ and number of photons in the pulse is 10^9.

The number of "kicks" for our system is limited by depolarization of atoms due to spontaneous emission. The scattering rate per one atom is equal to $\gamma_s = s\Gamma/2$, where $s = (\Gamma/2\Delta)^2(I/I_s)$ is the saturation parameter, I_s is the saturation intensity on resonsnce. (for 780 nm *Rb* transition $I_s \simeq 2 \cdot 10^{-3}W/cm^2$). After the simple calculations we obtain the following estimate for the number of scattered photons. $N_s = N_{at}(\Gamma^3/4\Delta^2)(W/I_0A) \simeq 0.5 \cdot 10^8$. This is comparable with the number of atoms in the sample. Therefore the sample will be depolarized after 1 "kick". In order to increase the number of "kicks" one need to increase the optical depth of the sample. It makes the experiment difficult. Required optical depth ($\kappa \sim 500$) can be achieved for example in the atomic BEC. It is probably possible to reduce the required optical depth if one for example place the atomic sample into the ring cavity.

In the rest of the paper we neglect the depolarization of spins and assume that our kicked top is ideal.

3 Poincare section

Lets consider for simplicity the following Floquet operator

$$F = e^{-i\beta F_z^2/2F} e^{-i\pi F_y/2}$$

that describes the rotation by an angle $\pi/2$ about the y axis followed by the nonlinear torsion about z axis. (It is always possible to adjust the time between pulses and magnetic field to obtain $\pi/2$ rotation along y axis.)

Since the number of atom is large and the changes of spin are much greater that the quantum uncertainty of spin one can consider the evolution of spin classically. In the classic limit $F >> 1$ the angular momentum vector is located on the sphere of radius F. A point on the sphere can be specified by angles θ and ϕ such as $F_x = F\sin\theta\cos\phi$, $F_y = F\sin\theta\cos\phi$, $F_z = F\cos\theta$.

It is convenient to work with the complex variable

$$\gamma = e^{i\phi}\tan\frac{\theta}{2} \tag{3}$$

According to the [1] in the classical limit the stroboscopic map of a point γ to its image Γ is

$$\Gamma = \frac{1+\gamma}{1-\gamma}\exp(-i\beta\frac{\gamma+\gamma^*}{1+\gamma\gamma^*}) \tag{4}$$

The example of the Poincare section for the $\beta = 3$ and $\beta = 8$ is shown on Fig. 1.

4 Periodic orbits

To find the classic periodic orbits we randomly choose the initial γ. Then the N iterations of the map (4) was performed. If the final point was close enough to the initial $((\Delta Re\ \gamma)^2+$

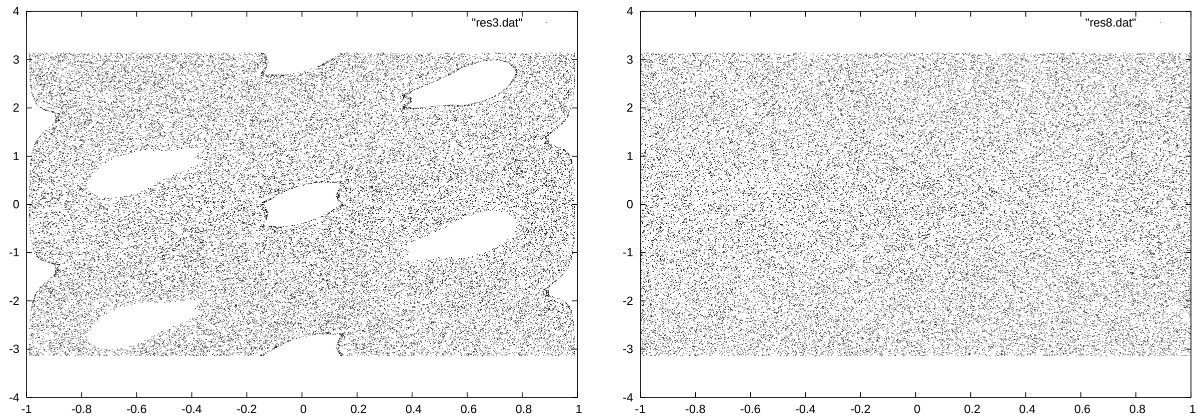

Figure 2: Poincare section for the kicked top. $k = 3$ (on the left) and $k = 8$ on the right. Vertical axis correspond to ϕ variable, horizontal axis is $\cos\theta$.

$(\Delta Im\, \gamma^2) < 1 \cdot 10^{-2}$), then the point was used as the initial guess and exact location of the periodic orbit was computed using the Newton method.

The points where the known periodic orbits crosses the Poincare plane for the periodic orbits of length 1, 2 and 3, are given in the Fig 2. Unfortunately due to lack of symbolic dynamic I can not count yet how many orbits I missed.

Stabilities of the periodic orbits were computed as well. The one time step Jacobian matrix is equal to

$$J_{ij} = \frac{\partial f_i(x)}{\partial x_j}$$

where f is $\{\Re\Gamma, \Im\Gamma\}$ and $x_j = \{\Re\gamma, \Im\gamma\}$. The total stability matrix for the periodic orbit with the period p was computed as the product of the single step matrices along the periodic orbit.

$$J^n = \prod_{n=1}^{p} J(x_n)$$

The stability of periodic orbits was calculated as the product of expanding eigenvalues of the stability matrix.

The result of our calculations for the shortest periodic orbits is given is the Table 1. I found 4 orbits of length 1, 5 orbit of length 2, 24 orbit of length 3, 103 orbits of length 4 and 328 orbits of length 5. The list of shortest periodic orbits is given in the Table 1.

5 Calculation of the average values of the spin variables

Once the periodic orbits are found, it is possible to evaluate the average values of the spin observable using formulas [3]

$$\langle F_i \rangle = \frac{\langle F_i \rangle_\zeta}{\langle n \rangle_\zeta} \tag{5}$$

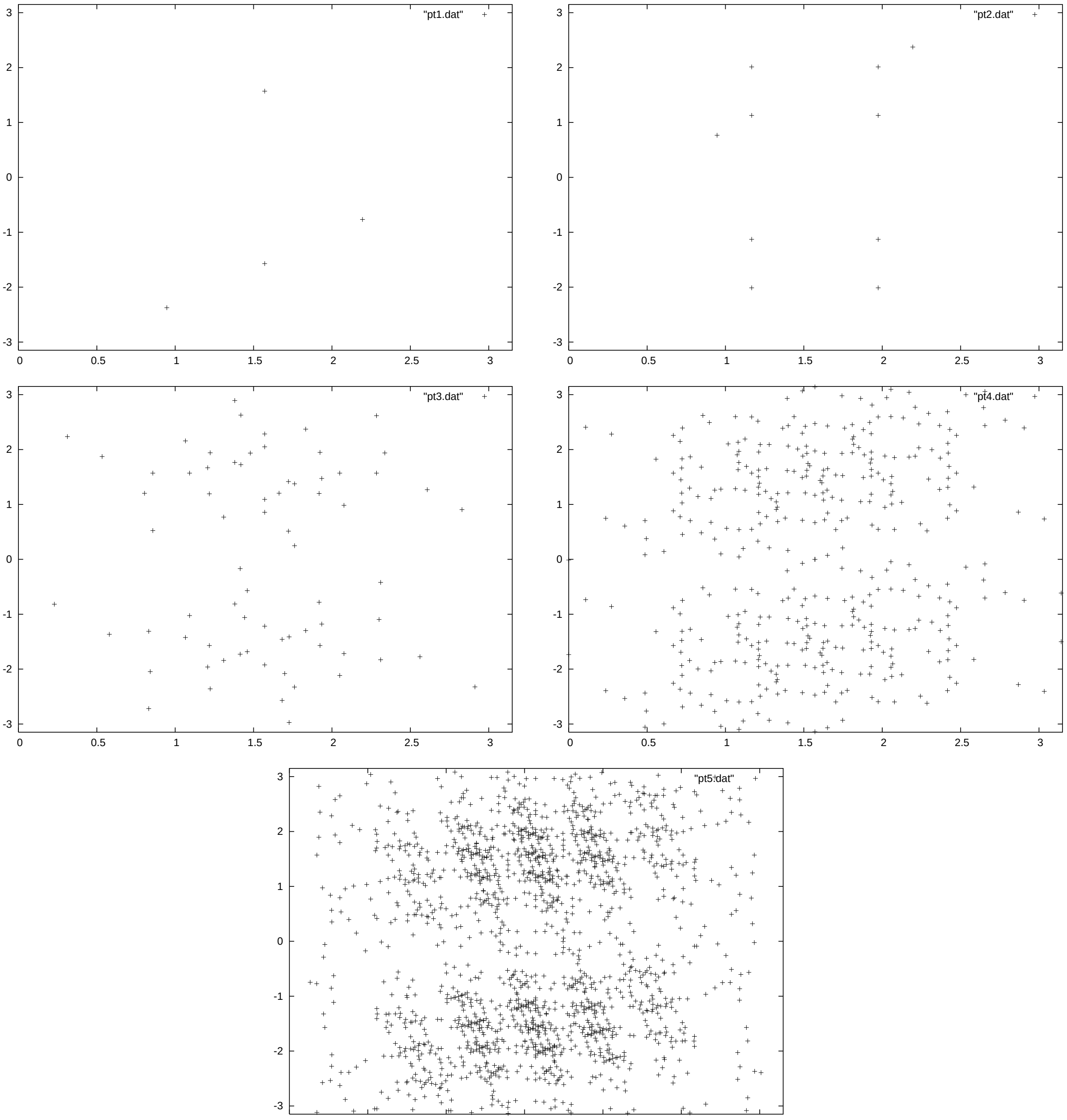

Figure 3: Point on the map that corresponds to the periodic orbits (orbit period is from 1 to 5). Vertical axis corresponds to ϕ variable, horizontal axis corresponds to θ.

Table 1: The list of the shortest periodic orbits for $k = 8$. θ_0 and ϕ_0 are the initial point for the periodic orbit.

Length	θ_0	ϕ_0	Stability, Λ_p
1	2.194924530160e+00	-7.667638962549e-01	5.35455654513e+00
1	9.466681249283e-01	-2.374828756595e+00	5.35455654611e+00
1	1.570796330867e+00	-1.570796327263e+00	7.87298334037e+00
1	1.570796323856e+00	1.570796327155e+00	7.87298335105e+00
2	9.466681234367e-01	7.667638962468e-01	2.86712757968e+01
2	1.167231719870e+00	-1.129622300318e+00	4.22371153380e+01
2	1.974360933785e+00	1.129622300295e+00	4.22371153398e+01
2	1.167231719870e+00	-2.011970353272e+00	4.22371153424e+01
2	1.167231719946e+00	1.129622300352e+00	4.22371155855e+01
3	3.121999576426e-01	2.235844272363e+00	3.60838499330e+01
3	1.761493163913e+00	2.487132361269e-01	3.60838499398e+01
3	2.292461972470e-01	-8.170371065824e-01	4.99142433840e+01
3	1.414640529515e+00	-1.728891648672e+00	4.99142434891e+01
3	1.722739256233e+00	1.417068438867e+00	6.45126263569e+01
3	5.337186412059e-01	1.872888521939e+00	6.45126303457e+01
3	5.801262195603e-01	-1.366815092740e+00	1.02208139060e+02
3	1.459532514395e+00	-1.682755310334e+00	1.02208139066e+02
3	8.310800973459e-01	-2.720320111434e+00	1.08623434186e+02
3	1.380493518200e+00	-8.141713442490e-01	1.08623434188e+02
3	8.572020146209e-01	1.570796326666e+00	1.52693090467e+02
3	1.570796328576e+00	2.284390639431e+00	1.52693090932e+02
3	2.076378738400e+00	9.841475922903e-01	2.21301802243e+02
3	1.065213914979e+00	2.157445061413e+00	2.21301804841e+02
3	1.223369815324e+00	-2.359091005214e+00	2.22264360495e+02
3	2.300678163664e+00	-1.096329985579e+00	2.22264360570e+02
3	1.309128982367e+00	7.698525299430e-01	2.45698034170e+02
3	1.832463671229e+00	2.371740123647e+00	2.45698034179e+02
3	1.091700833970e+00	-1.024575432185e+00	2.59242021587e+02
3	1.570796327499e+00	1.091700833800e+00	2.59242023196e+02
3	1.206726597751e+00	-1.961734887046e+00	3.69429470487e+02
3	1.662342284084e+00	1.205123760534e+00	3.69429470582e+02
3	1.570796326834e+00	-1.217961778762e+00	3.86481837100e+02
3	1.923630873811e+00	-1.570796326647e+00	3.86481841477e+02
...			

Table 2: The results of calculation of spin averages for $\beta = 8$.

Max. length	$\langle \hat{n} \rangle_\zeta$	$\langle \hat{F}_z \rangle / F$	$\langle \hat{F}_y \rangle / F$	$\langle \hat{F}_z^2 \rangle / F^2$	$\langle \hat{F}_y^2 \rangle / F^2$
1	0.627547	-0.210291	1.25138e-10	0.372428	0.127559
2	0.303125	-0.00526595	-7.80891e-12	0.0451172	0.0700529
3	0.749329	-0.0425953	-3.37247e-10	0.140823	0.0642145
4	2.14767	-0.00895313	6.41808e-11	0.0354662	0.0339362
5	0.20124	0.12999	-1.60777e-11	-0.185169	-0.069081

and

$$\langle F_i^2 \rangle = \frac{\langle F_i^2 \rangle_\zeta}{\langle n \rangle_\zeta} \tag{6}$$

where $\langle F_i^2 \rangle_\zeta$ and $\langle n \rangle_\zeta$ are the dynamical zeta function averages for the angular moment and length of periodic orbit.

$$\langle n \rangle_\zeta = \sum_p (-1)^{k+1} \frac{\hat{n}_{p1} + ... + \hat{n}_{pk}}{\Lambda_{p1} ... \Lambda_{pk}} \tag{7}$$

and

$$\langle F_i^j \rangle_\zeta = \sum_p (-1)^{k+1} \frac{(\hat{F}_{p1} + ... + \hat{F}_{pk})^j}{\Lambda_{p1} ... \Lambda_{pk}} \tag{8}$$

Here Λ_i is the orbit stability, F_i is the i-th component of angular momentum. In both formulas summation is over all distinct periodic orbits.

The results of this calculations are given in the Table 2. (Orbits that have stability $\Lambda = 1$ were excluded from calculations.)

I should note here that in the case of large parameter k (e. q. $k = 8$) one can probably guess the result just by looking at the Poincare section. The points on the Poincare section uniformly cover the surface of the sphere. Therefore the $\langle F_z \rangle$ and $\langle F_y \rangle$ should be approximately equal to zero.

Unfortunately my calculations does not confirm this guess for $\langle \hat{F}_z \rangle$. Moreover some of the results for $\langle F_{z,y}^2 \rangle$ are negative. One of the reason is low precision of calculation of the orbit stability for some orbits. (For example two orbits with the orbit period $n = 4$ involves points $\gamma = 1$ and $\gamma \to \infty$ where my calculations of Jacobian matrix include terms like 0/0. These points need special treatment and I do not have time to do it.) Another possible reason of the slow convergence is the presence of stable or marginally stable periodic orbits that can indicate intermittency. And finally, I don't know the symbolic dynamic for the kicked top. Therefore I cannot verify that I found all periodic orbits.

6 Conclusion

We have considered here only one of many possibilities to implement the chaotic system using the atomic spins. Another approach to implement the quantum kicked top for single atom with the spin $f > 1/2$ using the start shift in the linearly polarized light wave is discussed in [5].

The chaotic dynamics can be also observed in the system similar to one used in the resent spin squeezing experiment [4]. In this case to the magnetic field applied to the sample of atoms depend on rotation of polarization plane of linearly polarized light propagating through this sample.

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