Ph. D. Thesis
Local Structures in Extended Systems

Vachtang Putkaradze

May 8, 1997

Contents

Chapter 0

About this thesis

0.1 Goals and structure of the work

The material of this thesis is based on the research performed together with T. Bohr, S. Watanabe, J. Krug, P. Cvitanović and F. Christiansen. This research has resulted in a number of papers. I have chosen to present the articles in the thesis as they are: so much work has been invested in polishing their text, that I felt that I had no right to rewrite these articles with my own words. Also, by presenting these papers in full in this thesis I would like to give credit to my collaborators. However, I felt that for non-specialists in the field (and especially for students just beginning to study the topic), these articles alone might seem a little difficult to comprehend. Therefore, I have preceded each article by a general introduction to the topic. This thesis can be divided into three major parts.

The first part of the thesis is devoted to hydrodynamics (boundary layer flows with a free surface) and takes more then half of all the space (chapters 1-5). It might seem a little surprising, considering that our preprint [1] (presented in chapter 4) takes only four pages. However, I felt that [1] is so condensed that it would be very difficult for a non-specialist in the field to read it. I have thus devoted chapters 1-3 to the derivation of the results presented in [1]. I start with a general introduction to the boundary layer theory and the phenomenon of separation (chapter 1). I have tried to make the description quite elementary, emphasising the physics of the phenomenon. However, as always in fluid dynamics texts, one cannot get away without a certain number of equations, but I have tried to reduce this number to a minimum. After this introductory chapter 1, the integral methods are developed for the circular hydraulic jump in chapter 2 and for flows running down an inclined plane in chapter 3. These integral methods form the core of the hydrodynamic part of the thesis. Our preprint [1], as I have already mentioned, is presented in chapter 4. Further extensions of integral methods, applied to two-dimensional channel flows, are described in chapter 5.

The second main topic in this thesis is the derivation of global bounds for solutions of partial differential equations and the study of the singularities appearing in the solution. An introduction to the subject is presented in chapter 6, and the paper [2] is included in chapter 7. I have tried to make chapter 6 appealing for a more general audience: the method of multiple-scale analysis and intermediate asymptotics explained therein is frequently used in many areas of physics, and may also be of interest to a reader who is interested in applying these methods to different topics.

The third main topic is the description of spatio-temporal chaos in terms of periodic orbits. The introductory chapter 8 explains how to compute averages of a chaotic system in terms of periodic orbits. Our paper [3] is included in chapter 9. Our approach is quite general and is not limited to the Kuramoto-Sivashinsky equation, which we have studied as a paradigm of a chaotic system. Therefore, these chapters 8 and 9 may be of interest to the reader studying the long-term properties of deterministic spatio-temporally chaotic systems.

This thesis has an introductory character and is mainly addressed to students starting at the bachelor level (3-4th year), who wish to get involved in the areas described in the thesis. All chapters in the thesis require a certain acquaintance with the theory of ordinary and partial differential equations in given in a one-year course. Knowledge of the following topics will be helpful in reading the thesis:

1. For the hydrodynamic part (chapters 1-5), the reader should possess at least some elementary knowledge of hydrodynamics. A half-year course in hydrodynamics should be enough. To read chapter 3, the reader should be familiar with the notion of a dispersion relation.

2. To read chapters 6-7, it would be helpful for the reader to know some elementary facts from functional analysis (different functional spaces, no more then simple definitions). Reading the first chapter of any book on analysis should be enough.

3. To read chapters 8-9, the reader should be familiar with the theory of chaotic motions (attractors, Poincaré planes, periodic orbits, averages etc.) A one-year course in the subject should provide the reader with enough expertise.

0.2 On the notation used in this thesis

I have tried to use variables consistently in the chapters of the thesis which are devoted to the same topic. However, in different parts some notation may overlap. For example, the letter q in chapters 1-5 usually denotes flux, and in chapter 8 it means a vector quantity to be averaged. I hope that this overlap of notation will not lead to any confusion, since the topics of these chapters are sufficiently far apart.

I have chosen to enumerate systems of equations by a single number. If the first equation of the system is referenced, I put an additional number in the parentheses after a dot. For example, the notation (3.5.1) denotes a reference to the first equation of system 5 in chapter 3.

0.3 Acknowledgements

I would like to thank my collaborators and co-authors Tomas Bohr, Shinya Watanabe, Joachim Krug, Predrag Cvitanović and Freddy Christiansen, who made this thesis possible. The credit for all the results we have achieved should be shared with them, but I bear full responsibility for the flaws of this thesis. The guidance of my advisor Tomas Bohr was indispensable for me. I have learned a lot (not only about physics) from long and fruitful discussions with Peter Dimon, Mogens H. Jensen and Edward A. Spiegel. Wonderful experiments done by P. Dimon, C. Ellegaard, A. E. Hansen and A. Haaning have created a foundation for the hydrodynamic part of the thesis. Illuminating discussions with my friends and colleagues Niall D. Whelan, Stephen Creagh, Martin van Hecke, Kai

Hansen, K. H. Andersen and Jonas L. Hansen have enriched me with new inspiring ideas and have clarified my own misunderstanding of certain areas. I am deeply grateful to M. van Hecke, M. Howard, K. Sardemann and S. Watanabe for reading the draft version of the thesis. Finally, I would like to thank all my colleagues in the Center for Chaos and Turbulence studies (CATS) for the wonderful atmosphere I was enjoying there. My study was supported by CATS and Novo Nordisk Fonden, to whom I am greatly thankful.

Bibliography

[1] Bohr, T., Putkaradze, V & Watanabe, S. *Averaging theory for the structure of hydraulic jumps and separation in laminar free surface flows*, submitted to *Phys. Rev.Lett.*

[2] Putkaradze, V., Bohr, T. & Krug, J. (1997) *Global estimates and shocks for the noiseless conserved KPZ equation*, *Nonlinearity*, **10**, *1*, 1.

[3] Christiansen, F., Cvitanović & P. Putkaradze, V., (1997) *Hopf's last hope: Spatiotemporal chaos in terms of unstable recurrent patterns*, *Nonlinearity*, **10**, *1*, 55.