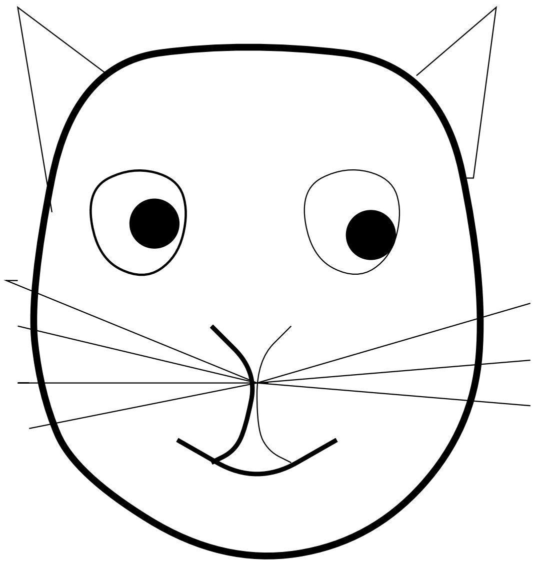
# Spatiotemporal chaos in terms of unstable spatiotemporally periodic states

Vachtang Putkaradze
Center For Chaos and Turbulence Studies, Niels Bohr Institute
Blegdamsvej 17, 2100 Copenhagen

May 6, 1996



*Exam Project for the course "Classical and Quantum Chaos"*
*Supervisor: Predrag Cvitanović*

**Abstract**

We describe spatiotemporally chaotic dynamics of a Kuramoto-Sivashinsky system by means of an infinite hierarchy of its unstable spatiotemporally periodic solutions. We construct an intrinsic parametrization of the corresponding strange set which serves as accurate guide to the high-dimensional dynamics, and evaluate several global averages characterizing the chaotic dynamics.

In recent years periodic orbits have been shown to be an effective tool in description of deterministic dynamical systems of low intrinsic dimension [1], in diagnosing deterministic chaos in noisy biological systems [2], and many other applications. In this report we apply the periodic orbit theory to the Kuramoto-Sivashinsky equation [3] in order to illustrate its utility in the description of spatially extended systems as well.

In what follows we shall refer to a periodic solution of dynamical equations as a "cycle", and to the closure of the union of all periodic solutions as the "strange set". Periodic

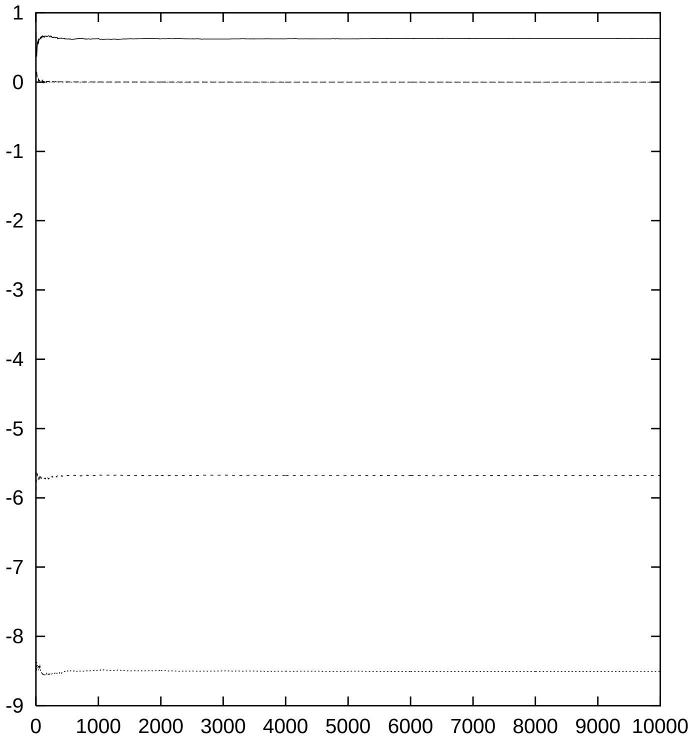



Figure 15: The value of the first 4 Lyapunov exponents for $N = 16$, chaotic regime $\nu = 0.02991$.

## 6 Direct Numerical Estimate of the Lyapunov Exponents

A method for the direct computation of the Lyapunov exponent was recently proposed by F. Christiansen and H.H.Rugh [20]. On the figure 15 we show the application of this method to our system, kindly provided to us by Freddy Christiansen. The truncation order was $N = 16$ and the value of viscosity $\nu = 0.02991$. The first Lyapunov exponent converges to the value $\lambda_1 = 0.62 \pm 0.01$. As follows from table 6, the value of the first Lyapunov exponent, computed from the periodic orbit theory, converges to the value 0.64. The escape rate is around 0.02 instead of being 0, so we can assert that, within the given accuracy, the direct numerical computation justifies the results obtained by the periodic orbit theory.

## 7 A Guide to Using Our Programs and Data Files.

This exam project required some extensive computer work and resulted in quite an amount of fortran programs and data files. We shall be happy if anyone could make use of our fortran codes for the sake of application to their specific problems or just for checking our results.

All the program, data and tex files are on the machine alf.nbi.dk. If you do not have an FTP access to this machine, please write an e-mail to putkarad@alf.nbi.dk-I will send you the files by e-mail. In the nearest future, the programs, tex and figures will be available on the Internet- check

http://WWW.nbi.dk/~predrag/QCcourse/.

The figures showing the Feigenbaum trees are not included due to their size. A

Table 1: The structure of the data file cyc**name**

| $l$ | $s_1$ | $\dots$ | $s_l$ | Topological length $l$ and symbolic sequence |
|---|---|---|---|---|
| $y_2^1$ | $\dots$ | $y_N^1$ | | $l$ Periodic points for the given orbit |
| | $\dots$ | | | |
| $y_2^l$ | $\dots$ | $y_N^l$ | | |
| | | $T$ | | Period of the orbit |
| | | $\Lambda_1$ | | Stability coefficients |
| | | $\dots$ | | |
| | | $\Lambda_N$ | | |

postscript version of the pictures takes several megabytes. Connect to

http://WWW.nbi.dk/~predrag/QCcourse/

to obtain the gif version of the figures (they are much smaller then the postscript files but cannot be included in the tex file).

All the data files have names which are self-explanatory. For example, if a file is called something16.3window.dat, then it corresponds to $N = 16$ dimensions, 3-window case. If a file is called something16.chaotic.dat, then it corresponds to the same system, but the chaotic regime. Hence, we write files as file**name**, where **name** can be equal to 9.chaotic.dat, 16.chaotic.dat, 17.chaotic.dat, 9.3window.dat or 16.3window.dat. All the data file are contained in the directory

/home/cmplx/putkarad/perorb/16dimensions/fredholm.

Data files available:

- Escape rate $\gamma$ is written into the files zero**name**.
- All $N$ Lyapunov exponents can be found in the files lyapfredh**name**.
- The values of the coefficients of the Fredholm determinant are in coeff**name**. NB: These data do not exist for all cases.
- Values of the Fredholm determinant $F(s, 0)$ as a function of $s$ are in values**name**.
- The periodic orbit data contained in the files cyc**name**. Given that the dimension of the phase space is $N$, the structure of the data file is represented in the table 1.

Fortran programs available:

- Program

  /home/cmplx/putkarad/perorb/16dimensions/fredholm/fredholm.f

  Computation of the Fredholm determinant and Lyapunov exponents. Reads from the file cyc**name** and writes into the files zero**name**, values**name**, lyapfredh**name** and coeff**name** (optional). Requires NAG library (root-finding).

- Program

  /home/cmplx/putkarad/perorb/16dimensions/fredholm/makeparametric.f.

  Constructs the parametric map $s \to f(s)$ and the time ceiling function $T(s)$. Reads from the file cyc**name** and writes into the file parametric.dat. Requires NAG (Runge-Kutta integrator) to compute the return times.

- Program

  /home/cmplx/putkarad/perorb/returnmap.f

  Computes the return map figure 6 for the given coordinate (parameter icoord in the program). Writes into the file returnmap.dat. Requires NAG (Runge-Kutta integrator).

- We have originally used the program

  /home/cmplx/putkarad/perorb/newtonall.f

  to compute the periodic orbits by method of Newton iteration (requires NAG- multi-dimensional root-finding and eigenvalues of a matrix). However, since this program uses only one-point shooting method, we would not recommend to use it. This program requires very accurate approximation to the initial point and may have some difficulties in computing periodic orbits with the period larger then 6. A much more advanced program, based on a multi-point shooting method, was written by Freddy Christiansen. It requires larger memory resources but is more robust with respect to the initial approximation.

  File /home/cmplx/freddy/putk/full.f

  Again, requires NAG: multi-dimensional root-finding and eigenvalues of a matrix.

- Program /home/cmplx/putkarad/perorb/profiles/makeprofile.f

  Computes the transforms of the periodic orbits in real space. Optionally, it can compute the differences between the periodic orbits. Reads from the file cyc**name** and writes into the file diff.dat. Requires NAG (Runge-Kutta integrator).

Please feel free to e-mail any comments, corrections or errors in the programs to putkarad@alf.nbi.dk

## Conclusions

We have constructed the symbolic dynamics and found the periodic orbits for an extended system -the Kuramoto-Sivashinsky equation. The method of averaging by means of the periodic orbits produced best results when the complete symbolic dynamics was known, i.e., for the period 3 window case. We can not claim that this method is computationally much better then the direct numerical simulation. A program to find periodic orbits up to length 10 for one value of viscosity had to run for one day of CPU on a dec-alfa workstation, which is much longer then the time required for the direct numerical simulation. Therefore, we shall make a conclusion that periodic orbit theory is , in this case, not efficient when the same results could be obtained by a direct simulation. However, periodic orbit theory allows to compute results otherwise not achievable, as Lyapunov exponents on the repeller. This seems to be a general property of all complicated devices: a microscope, which is not very efficient to hammer nails with, can be successfully used for much more refined work.

## 8 Acknowledgements

This project was done together with Predrag Cvitanovic and Freddy Christiansen. We share the achievements of this paper, however I assume full responsibility for its draw-