Dispersion management of cnoidal waves?

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Since the early 1980's there has been continuous development of optical communications technologies that employ solitons to encode information as light pulses and to transmit it over long distances through optical fibers. Recently researchers in the field of optical communications have discovered that enhanced power solitary waves can propagate for long distances in optical fibers when the dispersion of the fibers is modulated [1, 2, 3]. This technique has improved the transmission capacity of fiber communications lines.

A model of this technique is provided by the nonlinear Schrödinger equation (NLS) with periodic dispersion. A typical model would be

$$i\partial_z\psi + \alpha(z)\partial_t^2\psi + |\psi|\psi = 0 \ , \tag{1}$$

where we want the dispersion parameter α to vary with the propagation distance z, so

$$\alpha(z) = \begin{cases} \alpha_1 & 0 \le z < l_1 \\ \alpha_2 & l_1 \le z < l_2 \end{cases} \tag{2}$$

In communications devices this so-called dispersion map $\alpha(z)$ is repeated periodically many many times. On average, the light that travels down the fiber is confined to soliton-like pulses called dispersion managed solitons.

Notice that the NLS has solutions of the form $\psi = \phi(t)e^{i\lambda z}$, where $\phi(t)$ is a solution of

$$\alpha(z)\partial_t^2\phi - \lambda\phi + |\phi|\phi = 0 \ . \tag{3}$$

This equation can be reduced to quadratures for either value of the parameter $\alpha(z)$, so for simple solutions of the NLS you can compute the map over each segment l_1 and l_2. Call these F_1 and F_2.

We want to consider simple solutions of (3) and answer the following questions:

1. Given the maps F_i and the connections between them, are there any periodic orbits? That is can you propagate a solution to 3 down the fiber with the dispersion map $\alpha(z)$?

2. For how many iterations of the map does the initial solution persist? (You may construct a measure of the distance between the current solution and the initial data.) That is if there is a periodic orbit how robust is it to small variations?

There is a very simple way to do this [4]. The idea is to use a gaussian ansatz and study the modulation equations for its parameters. They find the return map and study in the SIAM paper available at the above web site.

References

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