Chapter 1

Cardioid Billiard

Spring Quarter 1997 Advanced Chaos Course

EXAM PROJECTS

The essence of this subject is incommunicable in print; the only way to developed intuition about chaotic dynamics is by computing, and you are urged to try to work through the essential steps in a project that combines the techniques learned in the course with some application of interest to you for other reasons. It is OK to share computer programs and such, but otherwise each project should be distinct, not a group project. The essential steps are:

- **Dynamics**
  1. construct a symbolic dynamics
  2. count prime cycles
  3. prune inadmissible itineraries, construct Markov graphs if appropriate
  4. implement a numerical simulator for your problem
  5. compute a set of the shortest periodic orbits
  6. compute cycle stabilities
- **Averaging, numerical**
  1. estimate by numerical simulation some observable quantitie, like the escape rate,
  2. or check the flow conservation, compute something like the Lyapunov exponent
- **Averaging, periodic orbits**
  1. implement the appropriate cycle expansions
  2. check flow conservation as function of cycle length truncation, if the system is closed

3. implement desymmetrization, factorization of zeta functions, if dynamics possesses a discrete symmetry
4. compute a quantity like the escape rate as a leading zero of a spectral determinant or a dynamical zeta function.
5. or evaluate a sequence of truncated cycle expansions for averages, such as the Lyapunov exponent or/and diffusion coefficients
6. compute a physically intersting quantity, such as the conductance
7. compute some number of the classical and/or quantum eigenvalues, if appropriate

1.1 Sacha Erner's project

Arnd Bdcker (arnd.baecker@physik.uni-ulm.de) revised 31 Oct 96
`http://www.physik.uni-ulm.de/theo/qc/baec/baecker.html`

1. Poincaré section
2. billiard map
3. conjugate points (optional)
4. symbolic dynamics a) stable-unstable mainfolds,
   b) partition line, 2 symbols
5. binary interpretation
6. plot long orbit, show pruned area
7. approximate grammars, Markov diagrams
8. estimate topological entropy
9. periodic orbits (perhaps by Kai Hansen's method,
   `http://www.nbi.dk/ predrag/QCcourse/orbits.ps.gz`)
10. a) Lyapunov by numerical averging
    b) Lyapunov by cycling
11. Fredholm determinant zeros vs. published eigenvalues

References

[1.1] A. Bdcker and H.R. Dullin, *"Symbolic dynamics and periodic orbits for the cardioid billiard"*, DESY report 95-198, chao-dyn 9511004 (1995)

[1.2] H. Bruus and N.D. Whelan, *"Edge diffraction, trace formulae and the cardioid billiard"*, *Nonlinearity* **9**, 1023 (1996), chao-dyn/9509005.

[1.3] A. Bdcker, F. Steiner and P. Stifter, *"Spectral statistics in the quantized cardioid billiard"*, *Phys. Rev.* **E 52**, 2463 (1995), DESY report 94-213, chao-dyn 9412007 (1994)

1.2 Other possible term projects

1. Lagrangian flow, passive advection
2. thermostat (stat. mech from deterministic chaos) (from L. Rondoni??)
3. 2-colliding disks gas
4. conductivity

5. triangular Lorentz gas
6. 4-disk repeller
7. ?? (open for suggestions)