Exercises

Exercise F.1 Lefschetz zeta function. *Elucidate the relation between the topological zeta function and the Lefschetz zeta function.*

Exercise F.2 Counting the 3-disk pinball counterterms. *Verify that the number of terms in the 3-disk pinball curvature expansion (15.31) is given by*

$$\begin{aligned}\prod_p (1+t_p) &= \frac{1-3z^4-2z^6}{1-3z^2-2z^3} = 1+3z^2+2z^3+\frac{z^4(6+12z+2z^2)}{1-3z^2-2z^3} \\ &= 1+3z^2+2z^3+6z^4+12z^5+20z^6+48z^7+84z^8+184z^9+\ldots \end{aligned} \qquad \text{(F.3)}$$

This means that, for example, c_6 has a total of 20 terms, in agreement with the explicit 3-disk cycle expansion (15.32).

Exercise F.3 Cycle expansion denominators**. *Prove that the denominator of c_k is indeed D_k, as asserted (E.14).*

Exercise F.4 Counting subsets of cycles. *The techniques developed above can be generalized to counting subsets of cycles. Consider the simplest example of a dynamical system with a complete binary tree, a repeller map (9.10) with two straight branches, which we label 0 and 1. Every cycle weight for such map factorizes, with a factor t_0 for each 0, and factor t_1 for each 1 in its symbol string. The transition matrix traces (10.5) collapse to $tr(T^k) = (t_0+t_1)^k$, and $1/\zeta$ is simply*

$$\prod_p (1-t_p) = 1-t_0-t_1 \qquad \text{(F.4)}$$

Substituting into the identity

$$\prod_p (1+t_p) = \prod_p \frac{1-t_p{}^2}{1-t_p}$$

we obtain

$$\begin{aligned}\prod_p (1+t_p) &= \frac{1-t_0^2-t_1^2}{1-t_0-t_1} = 1+t_0+t_1+\frac{2t_0t_1}{1-t_0-t_1} \\ &= 1+t_0+t_1+\sum_{n=2}^{\infty}\sum_{k=1}^{n-1} 2\binom{n-2}{k-1} t_0^k t_1^{n-k} . \end{aligned} \qquad \text{(F.5)}$$

Hence for $n \geq 2$ the number of terms in the expansion ?! with k 0's and $n-k$ 1's in their symbol sequences is $2\binom{n-2}{k-1}$. This is the degeneracy of distinct cycle *eigenvalues in fig.?!; for systems with non-uniform hyperbolicity this degeneracy is lifted (see fig. ?!).*

In order to count the number of prime cycles in each such subset we denote with $M_{n,k}$ $\quad (n = 1, 2, \ldots ; k = \{0, 1\}$ *for* $n = 1$; $k = 1, \ldots, n-1$ *for* $n \geq 2)$ *the number of prime* n*-cycles whose labels contain* k *zeros, use binomial string counting and Möbius inversion and obtain*

$$
\begin{aligned}
M_{1,0} &= M_{1,1} = 1 \\
nM_{n,k} &= \sum_{m \left| \frac{n}{k} \right.} \mu(m) \binom{n/m}{k/m}, \quad n \geq 2\,, k = 1, \ldots, n-1
\end{aligned}
$$

where the sum is over all m *which divide both* n *and* k.