Exercises

Exercise 13.1 What space does $\mathcal{L}$ act on? *Show that (13.2) is a complete basis on the space of analytic functions on a disk (and thus that we found the* complete *set of eigenvalues).*

Exercise 13.2 What space does $\mathcal{L}$ act on? *What can be said about the spectrum of (13.1) on $L^1[0,1]$? Compare the result with fig. 13.1.*

Exercise 13.3 Euler formula. *Derive the Euler formula (13.4)*

$$\prod_{k=0}^{\infty}(1+tu^k) = 1+\frac{t}{1-u}+\frac{t^2u}{(1-u)(1-u^2)}+\frac{t^3u^3}{(1-u)(1-u^2)(1-u^3)}\cdots$$

$$= \sum_{k=0}^{\infty} t^k \frac{u^{\frac{k(k-1)}{2}}}{(1-u)\cdots(1-u^k)}, \qquad |u|<1. \tag{13.26}$$

Exercise 13.4 2-d product expansion**. *We conjecture that the expansion corresponding to (13.26) is in this case*

$$\prod_{k=0}^{\infty}(1+tu^k)^{k+1} = \sum_{k=0}^{\infty}\frac{F_k(u)}{(1-u)^2(1-u^2)^2\cdots(1-u^k)^2}t^k$$

$$= 1+\frac{1}{(1-u)^2}t+\frac{2u}{(1-u)^2(1-u^2)^2}t^2 + \frac{u^2(1+4u+u^2)}{(1-u)^2(1-u^2)^2(1-u^3)^2}t^3+\cdots \tag{13.27}$$

$F_k(u)$ is a polynomial in u, and the coefficients fall off asymptotically as $C_n \approx u^{n^{3/2}}$. Verify; if you have a proof to all orders, e-mail it to the authors. (See also solution 13.3).

Exercise 13.5 Bernoulli shift on L spaces. *Check that the family (13.12) belongs to $L^1([0,1])$. What can be said about the essential spectral radius on $L^2([0,1])$? A useful reference is [13.28].*

Exercise 13.6 Cauchy integrals. *Rework all complex analysis steps used in the Bernoulli shift example on analytic functions on a disk.*

Exercise 13.7 Escape rate. *Consider the escape rate from a strange repeller: find a choice of trial functions ξ and φ such that (13.21) gives the fraction on particles surviving after n iterations, if their initial density distribution is $\rho_0(x)$. Discuss the behavior of such an expression in the long time limit.*