Exercises

Exercise 16.1 **Escape rate of the logistic map.**

(a) *Calculate the fraction of trajectories remaining trapped in the interval* $[0, 1]$ *for the logistic map*

$$f(x) = a(1 - 4(x - 0.5)^2), \tag{16.22}$$

and determine the a *dependence of the escape rate* $\gamma(a)$ *numerically.*

(b) *Work out a numerical method for calculating the lengths of intervals of trajectories remaining stuck for* n *iterations of the map.*

(c) *What is your expectation about the* a *dependence near the critical value* $a_c = 1$*?*

Exercise 16.2 **Four scale map decay.** *Compute the second largest eigenvalue of the Perron-Frobenius operator for the four scale map*

$$f(x) = \begin{cases} a_1 x & \text{if } 0 < x < b/a_1, \\ (1-b)((x - b/a_1)/(b - b/a_1)) + b & \text{if } b/a_1 < x < b, \\ a_2(x - b) & \text{if } b < x < b + b/a_2, \\ (1-b)((x - b - b/a_2)/(1 - b - b/a_2)) + b & \text{if } b + b/a_2 < x < 1. \end{cases} \tag{16.23}$$

Exercise 16.3 **Lyapunov exponents for 1-dimensional maps.** *Extend your cycle expansion programs so that the first and the second moments of observables can be computed. Use it to compute the Lyapunov exponent for some or all of the following maps:*

(a) the piecewise-linear flow conserving map, the skew tent map

$$f(x) = \begin{cases} ax & \text{if } 0 \le x \le a^{-1}, \\ \frac{a}{a-1}(1-x) & \text{if } a^{-1} \le x \le 1. \end{cases}$$

(b) the Ulam map $f(x) = 4x(1-x)$

(c) the skew Ulam map

$$f(x) = 0.1218x(1-x)(1-0.6x)$$

with a peak at 0.7*.*

(d) the repeller of $f(x) = Ax(1-x)$*, for either* $A = 9/2$ *or* $A = 6$ *(this is a continuation of exercise 15.2).*

(e) *for the 2-branch flow conserving map*

$$\begin{aligned} f_0(x) &= \frac{h - p + \sqrt{(h-p)^2 + 4hx}}{2h}, \qquad x \in [0, p] \\ f_1(x) &= \frac{h + p - 1 + \sqrt{(h+p-1)^2 + 4h(x-p)}}{2h}, \qquad x \in [p, 1] \end{aligned} \tag{16.24}$$

This is a nonlinear perturbation of ($h = 0$) Bernoulli map (13.10); the first 15 eigenvalues of the Perron-Frobenius operator are listed in ref. [16.1] for $p = 0.8$, $h = 0.1$. Use these parameter values when computing the Lyapunov exponent.

Cases (a) and (b) can be computed analytically; cases (c), (d) and (e) require numerical computation of cycle stabilities. Just to see whether the theory is worth the trouble, also cross check your cycle expansions results for cases (c) and (d) with Lyapunov exponent computed by direct numerical averaging along trajectories of randomly chosen initial points:

(f) *trajectory-trajectory separation (8.23) (hint: rescale δx every so often, to avoid numerical overflows),*

(g) *iterated stability (8.27).*

How good is the numerical accuracy compared with the periodic orbit theory predictions?