Exercises

Exercise 9.1 **Binary symbolic dynamics.** *Verify that the shortest prime binary cycles of the unimodal repeller of fig. 9.8 are* $\overline{0}$, $\overline{1}$, $\overline{01}$, $\overline{001}$, $\overline{011}$, $\cdots$. *Compare with table 9.2. Try to sketch them in the graph of the unimodal function* $f(x)$*; compare ordering of the periodic points with fig. 9.9. The point is that while overlayed on each other the longer cycles look like a hopeless jumble, the cycle points are clearly and logically ordered by the alternating binary tree.*

Exercise 9.2 **3-disk fundamental domain symbolic dynamics.** *Try to sketch* $\overline{0}$, $\overline{1}$, $\overline{01}$, $\overline{001}$, $\overline{011}$, $\cdots$. *in the fundamental domain, fig. 9.5, and interpret the symbols* $\{0, 1\}$ *by relating them to topologically distinct types of collisions. Compare with table 9.1. Then try to sketch the location of periodic points in the Poincaré section of the billiard flow. The point of this exercise is that while in the configuration space longer cycles look like a hopeless jumble, in the Poincaré section they are clearly and logically ordered. The Poincaré section is always to be preferred to projections of a flow onto the configuration space coordinates, or any other subset of phase space coordinates which does not respect the topological organization of the flow.*

Exercise 9.3 **Generating prime cycles.** *Write a program that generates all binary prime cycles up to given finite length.*

Exercise 9.4 **A contracting baker's map.** *Consider a contracting (or "dissipative") baker's defined in exercise 4.3.*

The symbolic dynamics encoding of trajectories is realized via symbols 0 $(y \le 1/2)$ *and* 1 $(y > 1/2)$. *Consider the observable* $a(x, y) = x$. *Verify that for any periodic orbit* p $(\epsilon_1 \ldots \epsilon_{n_p})$, $\epsilon_i \in \{0, 1\}$

$$A_p = \frac{3}{4} \sum_{j=1}^{n_p} \delta_{j,1} \, .$$

Exercise 9.5 **Reduction of 3-disk symbolic dynamics to binary.**

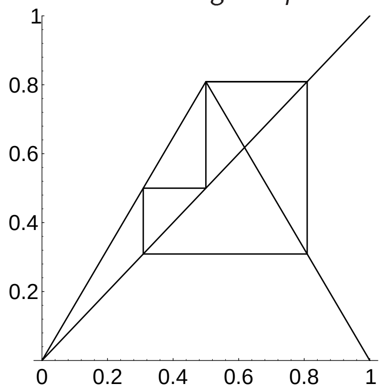
(a) *Verify that the 3-disk cycles*
$\{\overline{1\,2}, \overline{1\,3}, \overline{2\,3}\}$, $\{\overline{1\,2\,3}, \overline{1\,3\,2}\}$, $\{\overline{12\,13}$ + *2 perms.*$\}$,
$\{\overline{121\,232\,313}$ + *5 perms.*$\}$, $\{\overline{121\,323}$+ *2 perms.*$\}$, $\cdots$,
correspond to the fundamental domain cycles $\overline{0}$, $\overline{1}$, $\overline{01}$, $\overline{001}$, $\overline{011}$, $\cdots$ *respectively.*

(b) *Check the reduction for short cycles in table 9.1 by drawing them both in the full 3-disk system and in the fundamental domain, as in fig. 9.5.*

(c) *Optional: Can you see how the group elements listed in table 9.1 relate irreducible segments to the fundamental domain periodic orbits?*

Exercise 9.6 Unimodal map symbolic dynamics. *Show that the tent map point $\gamma(S^+)$ with future itinerary S^+ is given by converting the sequence of s_n's into a binary number by the algorithm (9.13). This follows by inspection from the binary tree of fig. 9.9.*

Exercise 9.7 "Golden mean" pruned map. *Consider a symmetrical tent map on the unit interval such that its highest point belongs to a 3-cycle:*



(a) *Find the absolute value Λ for the slope (the two different slopes $\pm\Lambda$ just differ by a sign) where the maximum at $1/2$ is part of a period three orbit, as in the figure.*

(b) *Show that no orbit of this map can visit the region $x > (1+\sqrt{5})/4$ more than once. Verify that once an orbit exceeds $x > (\sqrt{5}-1)/4$, it does not reenter the region $x < (\sqrt{5}-1)/4$.*

(c) *If an orbit is in the interval $(\sqrt{5}-1)/4 < x < 1/2$, where will it be on the next iteration?*

(d) *If the symbolic dynamics is such that for $x < 1/2$ we use the symbol 0 and for $x > 1/2$ we use the symbol 1, show that no periodic orbit will have the substring $\_00\_$ in it.*

(e) *On the second thought, is there a periodic orbit that violates the above $\_00\_$ pruning rule?*

For continuation, see exercise 10.6 and exercise 10.8. See also exercise 10.7 and exercise 10.9.

Exercise 9.8 Binary 3-step transition matrix. *Construct [8×8] binary 3-step transition matrix analogous to the 2-step transition matrix (9.14). Convince yourself that the number of terms of contributing to $\mathrm{tr}\, T^n$ is independent of the memory length, and that this [$2^m \times 2^m$] trace is well defined in the infinite memory limit $m \to \infty$.*