Exercises

Exercise 3.1 **Rössler system** *(continuation of exercise 2.7) Construct a Poincaré section for this flow. How good an approximation would a replacement of the return map for this section by a 1-dimensional map be?*

Exercise 3.2 **Arbitrary Poincaré sections.** *We will generalize the construction of Poincaré section so that it can have any shape, as specified by the equation* $g(x) = 0$.

(a) *Start out by modifying your integrator so that you can change the coordinates once you get near the Poincaré section. You can do this easily by writing the equations as*

$$\frac{dx_k}{ds} = \kappa f_k \,, \tag{3.14}$$

with $dt/ds = \kappa$, *and choosing* κ *to be* 1 *or* $1/f_1$. *This allows one to switch between* t *and* x_1 *as the integration "time."*

(b) *Introduce an extra dimension* x_{n+1} *into your system and set*

$$x_{n+1} = g(x) \,. \tag{3.15}$$

How can this be used to find the Poincaré section?

Exercise 3.3 **Classical collinear helium dynamics.** *(continuation of exercise 2.9)*

Make a Poincaré surface of section by plotting (r_1, p_1) *whenever* $r_2 = 0$: *Note that for* $r_2 = 0$, p_2 *is already determined by (5.3). Compare your results with fig. 29.3(b).*

(Gregor Tanner, Per Rosenqvist)

Exercise 3.4 **Hénon map fixed points.** *Show that the two fixed points* (x_0, x_0), (x_1, x_1) *of the Hénon map (3.10) are given by*

$$\begin{aligned} x_0 &= \frac{-(1-b) - \sqrt{(1-b)^2 + 4a}}{2a} \,, \\ x_1 &= \frac{-(1-b) + \sqrt{(1-b)^2 + 4a}}{2a} \,. \end{aligned} \tag{3.16}$$

Exercise 3.5 **How strange is the Hénon attractor?**

(a) *Iterate numerically some 100,000 times or so the Hénon map*

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 - ax^2 + y \\ bx \end{bmatrix}$$

for $a = 1.4$, $b = 0.3$. *Would you describe the result as a "strange attractor"? Why?*

(b) *Now check how robust the Hénon attractor is by iterating a slightly different Hénon map, with* $a = 1.39945219$, $b = 0.3$. *Keep at it until the "strange" attracttor vanishes like a smile of the Chesire cat. What replaces it? Would you describe the result as a "strange attractor"? Do you still have confidence in your own claim for the part (a) of this exercise?*

Exercise 3.6 Fixed points of maps. *A continuous function* F *is a contraction of the unit interval if it maps the interval inside itself.*

(a) *Use the continuity of* F *to show that a one-dimensional contraction* F *of the interval* $[0, 1]$ *has at least one fixed point.*

(b) *In a uniform (hyperbolic) contraction the slope of* F *is always smaller than one,* $|F'| < 1$. *Is the composition of uniform contractions a contraction? Is it uniform?*