Exercises

Exercise 4.1 **Trace-log of a matrix.** *Prove that*

$$\det M = e^{\mathrm{tr}\,\ln M}.$$

for arbitrary finite dimensional matrix M.

Exercise 4.2 **Stability, diagonal case.** *Verify the relation (4.14)*

$$\mathbf{J}^t = e^{t\mathbf{A}} = \mathbf{U}^{-1} e^{t\mathbf{A}_D} \mathbf{U}.$$

Exercise 4.3 **A contracting baker's map.** *Consider a contracting (or "dissipative") baker's map, on $[0,1]^2$, defined as*

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} x_n/3 \\ 2y_n \end{pmatrix} \qquad y_n \le 1/2$$

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = \begin{pmatrix} x_n/3 + 1/2 \\ 2y_n - 1 \end{pmatrix} \qquad y_n > 1/2$$

This map shrinks strips by factor 1/3 in the x direction, and stretches (and folds) by factor 2 in the y direction.

How fast does the phase space volume contract?