## Exercises

**Exercise 19.1** **Sawtooth map desymmetrization.** *Work out the some of the shortest global cycles of different symmetries and fundamental domain cycles for the sawtooth map of fig. 19.1. Compute the dynamical zeta function and the spectral determinant of the Perron-Frobenius operator for this map; check explicitely the factorization (19.2).*

**Exercise 19.2** **2-$d$ asymmetric representation.** *The above expressions can sometimes be simplified further using standard group-theoretical methods. For example, the* $\frac{1}{2}\left((\mathrm{tr}\, M)^2 - \mathrm{tr}\, M^2\right)$ *term in (19.16) is the trace of the antisymmetric part of the* $M \times M$ *Kronecker product; if* $\alpha$ *is a 2-dimensional representation, this is the* $A_2$ *antisymmetric representation, so*

$$\text{2-dim:} \quad \det\left(1 - D_\alpha(h)t\right) = 1 - \chi_\alpha(h)t + \chi_{A_2}(h)t^2. \tag{19.28}$$

**Exercise 19.3** **3-disk desymmetrization.**

a) *Work out the 3-disk symmetry factorization for the* $0$ *and* $1$ *cycles, i.e. which symmetry do they have, what is the degeneracy in full space and how do they factorize (how do they look in the* $A_1$, $A_2$ *and the* $E$ *representations).*

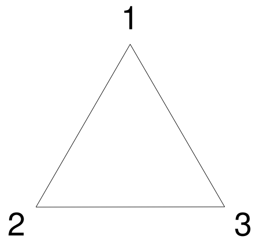
b) *Find the shortest cycle with no symmetries and factorize it like in a)*

c) *Find the shortest cycle that has the property that its time reversal is not described by the same symbolic dynamics.*

d) *Compute the dynamical zeta functions and the spectral determinants (symbolically) in the three representations; check the factorizations (19.15) and (19.17).*

*(Per Rosenqvist)*

**Exercise 19.4** **The group $C_{3v}$.** *We will compute a few of the properties of the group* $C_{3v}$, *the group of symmetries of an equilateral triangle*



**(a)** *All discrete groups are isomorphic to a permutation group or one of its subgroups, and elements of the permutation group can be expressed as cycles. Express the elements of the group $C_{3v}$ as cycles. For example, one of the rotations is $(123)$, meaning that vertex 1 maps to 2 and 2 to 3 and 3 to 1.*

**(b)** *Find the subgroups of the group $C_{3v}$.*

**(c)** *Find the classes of $C_{3v}$ and the number of elements in them.*

**(d)** *Their are three irreducible representations for the group. Two are one dimensional and the other one is formed by $2 \times 2$ matrices of the form*

$$\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} .$$

*Find the matrices for all six group elements.*

**(e)** *Use your representation to find the character table for the group.*

**Exercise 19.5** **$C_2$ factorizations: the Lorenz and Ising systems.** *In the Lorenz system [2.1, 19.15] the labels $+$ and $-$ stand for the left or the right lobe of the attractor and the symmetry is a rotation by $\pi$ around the $z$-axis. Similarly, the Ising Hamiltonian (in the absence of an external magnetic field) is invariant under spin flip. Work out the factorizations for some of the short cycles in either system.*

**Exercise 19.6** **Ising model.** *The Ising model with two states $\epsilon_i = \{+,-\}$ per site, periodic boundary condition, and Hamiltonian*

$$H(\epsilon) = -J \sum_i \delta_{\epsilon_i,\epsilon_{i+1}} ,$$

*is invariant under spin-flip: $+ \leftrightarrow -$. Take advantage of that symmetry and factorize the dynamical zeta function for the model, that is, find all the periodic orbits that contribute to each factor and their weights.*

**Exercise 19.7** **One orbit contribution.** *If $p$ is an orbit in the fundamental domain with symmetry $h$, show that it contributes to the spectral determinant with a factor*

$$\det\left(1 - D(h)\frac{t_p}{\lambda_p^k}\right) ,$$

*where $D(h)$ is the representation of $h$ in the regular representation of the group.*