Exercises

Exercise 17.1 Thermodynamics in higher dimensions *Introduce the time averages of the eigenvalues of the Jacobian*

$$\lambda_i = \lim_{t\to\infty} \frac{1}{t} \log |\Lambda_i^t(x_0)|, \tag{17.23}$$

as a generalization of (8.27).

Show that in higher dimensions Pesin's formula is

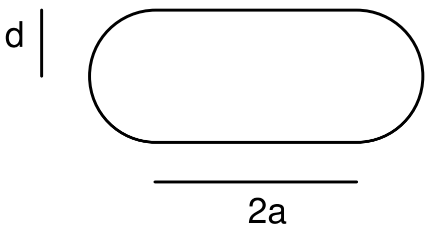
$$K_1 = \sum_i \lambda_i - \gamma, \tag{17.24}$$

where the summation goes for the positive λ_i-s only. (Hint: Use the higher dimensional generalization of (16.10)

$$\mu_i = e^{n\gamma}/|\prod_j \Lambda_{i,j}|,$$

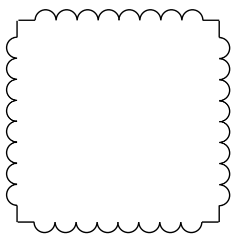
where the product goes for the expanding eigenvalues of the Jacobian of the periodic orbit.

Exercise 17.2 Bunimovich stadium Kolmogorov entropy. *Take for definitiveness $a = 1.6$ and $d = 1$ in the Bunimovich stadium of exercise 5.4,*



estimate the Lyapunov exponent by averaging over a very long trajectory. Biham and Kvale [17.14] estimate the discrete time Lyapunov to $\lambda \approx 1.0 \pm .1$, the continuous time Lyapunov to $\lambda \approx 0.43 \pm .02$, the topological entropy (for their symbolic dynamics) $h \approx 1.15 \pm .03$.

Exercise 17.3 Entropy of rugged-edge billiards. *Take a semi-circle of diameter ε and replace the sides of a unit square by $\lfloor 1/\varepsilon \rfloor$ catenated copies of the semi-circle.*



(a) *Is the billiard ergodic as $\varepsilon \to 0$?*

(b) *(hard) Show that the entropy of the billiard map is*

$$K_1 \to -\frac{2}{\pi} \ln \varepsilon + \text{const}\,,$$

as $\varepsilon \to 0$. (Hint: do not write return maps.)

(c) *(harder) Show that when the semi-circles of the Bunimovich stadium are far apart, say L, the entropy for the flow decays as*

$$K_1 \to \frac{2 \ln L}{\pi L}\,.$$

Exercise 17.4 **Two scale map** *Compute all those quantities - dimensions, escape rate, entropies, etc. - for the repeller of the one dimensional map*

$$f(x) = \begin{cases} 1 + ax & \textit{if} \quad x < 0, \\ 1 - bx & \textit{if} \quad x > 0. \end{cases} \tag{17.25}$$

where a and b are larger than 2. Compute the fractal dimension, plot the pressure and compute the $f(\alpha)$ spectrum of singularities.

Exercise 17.5 **Four scale map** *Compute the Rényi entropies and $g(\lambda)$ for the four scale map*

$$f(x) = \begin{cases} a_1 x & \textit{if} \quad 0 < x < b/a_1, \\ (1-b)((x - b/a_1)/(b - b/a_1)) + b & \textit{if} \quad b/a_1 < x < b, \\ a_2(x - b) & \textit{if} \quad b < x < b + b/a_2, \\ (1-b)((x - b - b/a_2)/(1 - b - b/a_2)) + b & \textit{if} \quad b + b/a_2 < x < 1. \end{cases} \tag{17.26}$$

Hint: Calculate the pressure function and use (17.13).

Exercise 17.6 **Transfer matrix** *Take the unimodal map $f(x) = \sin(\pi x)$ of the interval $I = [0, 1]$. Calculate the four preimages of the intervals $I_0 = [0, 1/2]$ and $I_1 = [1/2, 1]$. Extrapolate $f(x)$ with piecewise linear functions on these intervals. Find a_1, a_2 and b of the previous exercise. Calculate the pressure function of this linear extrapolation. Work out higher level approximations by linearly extrapolating the map on the 2^n-th preimages of I.*