Exercises

Exercise 27.1 Monodromy matrix from second variations of the action. *Show that*

$$D_{\perp j}/D'_{\perp j} = (\mathbf{1} - \mathbf{J}) \tag{27.19}$$

Exercise 27.2 Jacobi gymnastics. *Prove that the ratio of determinants in (O.27) can be expressed as*

$$\frac{\det D'_{\perp j}(q_\|, 0, E)}{\det D_{\perp j}(q_\|, 0, E)} = \det \begin{pmatrix} I - \mathbf{J}_{qq} & -\mathbf{J}_{qp} \\ -\mathbf{J}_{pq} & I - \mathbf{J}_{pp} \end{pmatrix} = \det (1 - \mathbf{J}_j)\,, \tag{27.20}$$

where $\mathbf{J}_j$ is the monodromy matrix of the periodic orbit.

Exercise 27.3 Volume of d-dimensional sphere. *Show that the volume of a d-dimensional sphere of radius r equals $\pi^{d/2} r^d / \Gamma(1 + d/2)$. Show that $\Gamma(1 + d/2) = \Gamma(d/2)d/2$.*

Exercise 27.4 Average density of states in 1 dimension. *Show that in one dimension the average density of states is given by (27.8)*

$$\bar{d}(E) = \frac{T(E)}{2\pi\hbar},$$

where $T(E)$ is the time period of the 1-dimensional motion and show that

$$\bar{N}(E) = \frac{S(E)}{2\pi\hbar}\,, \tag{27.21}$$

where $S(E) = \oint p(q, E)\,dq$ is the action of the orbit.

Exercise 27.5 Average density of states in 2 dimensions. *Show that in 2 dimensions the average density of states is given by (27.9)*

$$\bar{d}(E) = \frac{m\mathcal{A}(E)}{2\pi\hbar^2}\,,$$

where $\mathcal{A}(E)$ is the classically allowed area of configuration space for which $U(q) < E$.