Exercises

Exercise 26.1 **Dirac delta function, Lorentzian representation.** *Derive (23.18).*

Exercise 26.2 **Transport equations.** *Write the wave-function in the asymptotic form*

$$\psi(q,t) = e^{\frac{i}{\hbar}R(x,t)+\frac{i}{\hbar}\varepsilon t}\sum_{n\geq 0}(i\hbar)^n A_n(x,t)\,.$$

Derive the transport equations for the A_n by substituting this into the Schrödinger equation and then collecting terms by orders of $\hbar$. Notice that equation for $\dot{A}_n$ only requires knowledge of A_{n-1} and R.

Exercise 26.3 **Easy examples of the Hamilton's principal function.** *Calculate $R(q,q',t)$ for*

a) a d-dimensional free particle

b) a 3-dimensional particle in constant magnetic field

c) a 1-dimensional harmonic oscillator.

Continuation: (26.9).

Exercise 26.4 **Dirac delta function, gaussian representation.** *Consider the gaussian distribution function*

$$\delta_\sigma(z) = \frac{1}{\sqrt{2\pi\sigma^2}}e^{-z^2/2\sigma^2}\,.$$

Show that in $\sigma \to 0$ limit this is the Dirac delta function (26.24).

Exercise 26.5 **d-dimensional free particle propagator.** *Verify the results in sect. 26.2.2; show explicitely that (26.33), the semiclassical Van Vleck propagator in d dimensions, solves the Schrödinger's equation.*

Exercise 26.6 **Charged particle in constant magnetic field.** *Calculate the semiclassical propagator for a charged particle in constant magnetic field in 3 dimensions. Verify that the semiclassical expression coincides with the exact solution.*

Exercise 26.7 **1-dimensional harmonic oscillator propagator.** *Calculate the semiclassical propagator for a 1-dimensional harmonic oscillator and verify that it is identical to the exact quantum propagator.*

Exercise 26.8 **d-dimensional gaussian integrals.** *Show that the gaussian integral in d-dimensions is given by*

$$\frac{1}{(2\pi)^{d/2}} \int d^d x e^{-\frac{1}{2}x^T \cdot M^{-1} \cdot x + x \cdot J} \quad = \quad |\det M|^{\frac{1}{2}} \, e^{\frac{1}{2}J^T \cdot M \cdot J}\,, \qquad (26.52)$$

where M is a real positive definite $[d \times d]$ matrix, that is a matrix with strictly positive eigenvalues. x, J are d-dimensional vectors, on x^T is the transpose of x.

Exercise 26.9 **Free particle action.** *Calculate the energy dependent action for a free particle, a charged particle in a constant magnetic field and for the harmonic oscillator.*

Exercise 26.10 **Zero length orbits.** *Derive the classical trace (11.1) rigorously and either add the $t \to 0_+$ zero length contribution to the trace formula, or show that it vanishes. Send us a reprint of* Phys. Rev. Lett. *with the correct derivation.*

Exercise 26.11 **A usefull determinant identity.** *Show that the following two determinants equal each other:*
$A[(n+1) \times (n+?)] determinant$

$$\det(M'_n) \quad = \quad \begin{array}{cc} n+1 & \\ \left[\begin{array}{cccc} x_{1,1} & \dots & x_{1,n} & y_1 \\ \vdots & \ddots & \vdots & \vdots \\ x_{n,1} & \dots & x_{n,n} & y_n \\ z_1 & \dots & z_n & E \end{array} \right] & n+1 \end{array} \qquad (26.53)$$

and the $[n \times n]$ determinant:

$$E \det(M_n) = E \quad \begin{array}{cc} n & \\ \left[\begin{array}{ccc} x_{1,1} - y_1 z_1 E^{-1} & \dots & x_{1,n} - y_1 z_n E^{-1} \\ \vdots & \ddots & \vdots \\ x_{n,1} - y_n z_1 E^{-1} & \dots & x_{n,n} - y_n z_n E^{-1} \end{array} \right] & n \end{array} \qquad (26.54)$$

Exercise 26.12 **Free particle semiclassical Green's functions.** *Calculate the semiclassical Green's functions for the systems of exercise 26.9.*