Exercises

(N. Whelan)

Exercise 30.1 Stationary phase integral. *Evaluate the two stationary phase integrals corresponding to contours E_1 and E_2 of fig. 30.3 and thereby verify (30.7).*

Exercise 30.2 Scattering from a small disk *Imagine that instead of a wedge, we have a disk whose radius a is much smaller than the typical wavelengths we are considering. In that limit, solve the quantum scattering problem - find the scattered wave which result from an incident plane wave. You can do this by the method of partial waves - the analogous three dimensional problem is discussed in most quantum textbooks. You should find that only the $m = 0$ partial wave contributes for small a. Following the discussion above, show that the diffraction constant is*

$$d = \frac{2\pi}{\log\left(\frac{2}{ka}\right) - \gamma_e + i\frac{\pi}{2}} \tag{30.28}$$

where $\gamma_e = 0.577\cdots$ is Euler's constant. Note that in this limit d depends weakly on k but not on the scattering angle.

Exercise 30.3 Several diffractive legs. *Derive equation (30.18). The calculation involves considering slight variations of the diffractive orbit as in the simple case discussed above. Here it is more complicated because there are more diffractive arcs - however you should convince yourself that a slight variation of the diffractive orbit only affects one leg at a time.*

Exercise 30.4 Unsymmetrized dynamical zeta function. *Assume you know nothing about symmetry decomposition. Construct the three node Markov diagram for fig. 30.1 by considering A, B and B' to be physically distinct. Write down the corresponding dynamical zeta function and check explicitly that for $B = B'$ it factorizes into the product of the the even and odd dynamical zeta functions. Why is there no term $t_{\bar{2}}$ in the full dynamical zeta function?*

Exercise 30.5 Three point scatterers. *Consider the limiting case of the three disk game of pinball of fig. 1.1 where the disks are very much smaller than their spacing R. Use the results of exercise 30.2 to construct the desymmetrized dynamical zeta functions, as in sect. 19.6. You should find $1/\zeta_{A_1} = 1 - 2t$ where $t = de^{i(kR-3\pi/4)}/\sqrt{8\pi kR}$. Compare this formula with that from chapter 9. By assuming that the real part of k is much greater than the imaginary part show that the positions of the resonances are $k_nR = \alpha_n - i\beta_n$ where $\alpha_n = 2\pi n + 3\pi/4$, $\beta_n = \log\left(\sqrt{2\pi\alpha_n}/d\right)$ and n is a non-negative integer. (See also reference [30.11].)*