Exercises

Exercise 7.1 Coordinate transformations. *Changing coordinates is conceptually simple, but can become confusing when carried out in detail. The difficulty arises from confusing functional relationships, such as $x(t) = h^{-1}(y(t))$ with numerical relationships, such as $w(y) = h'(x)v(x)$. Working through an example will clear this up.*

(a) *The differential equation in the $\mathcal{M}$ space is $\dot{x} = \{2x_1, x_2\}$ and the change of coordinates from $\mathcal{M}$ to $\mathcal{M}'$ is $h(x_1, x_2) = \{2x_1 + x_2, x_1 - x_2\}$. Solve for $x(t)$. Find h^{-1}.*

(b) *Show that in the transformed space $\mathcal{M}'$, the differential equation is*

$$\frac{d}{dt}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 5y_1 + 2y_2 \\ y_1 + 4y_2 \end{bmatrix} . \tag{7.26}$$

Solve this system. Does it match the solution in the $\mathcal{M}$ space?

Exercise 7.2 Linearization for maps. *Let $f : C \to C$ be a map from the complex numbers into themselves, with a fixed point at the origin and analytic there. By manipulating power series, find the first few terms of the map h that conjugates f to αz, that is,*

$$f(z) = h^{-1}(\alpha h(z)) .$$

There are conditions on the derivative of f at the origin to assure that the conjugation is always possible. Can you formulate these conditions by examining the series?

(difficulty: medium)

Exercise 7.3 Ulam and tent maps. *Show that the smooth conjugacy (7.1)*

$$g(y_0) = h \circ f \circ h^{-1}(y_0) , \qquad y = h(x) = \sin^2(\pi x/2) ,$$

conjugates the tent map $f(x) = 1 - 2|x - 1/2|$ into the Ulam map $g(y) = 4y(1 - y)$. (Continued as exercise 12.1.)