Exercises

Exercise 3.1 **Poincaré sections of the Rössler flow.** *(continuation of exercise 2.8) Calculate numerically a Poincaré section (or several Poincaré sections) of the Rössler flow. As the Rössler flow state space is 3-dimensional, the flow maps onto a 2-dimensional Poincaré section. Do you see that in your numerical results? How good an approximation would a replacement of the return map for this section by a 1-dimensional map be? More precisely, estimate the thickness of the strange attractor. (continued as exercise 4.4)*

(Rytis Paškauskas)

Exercise 3.2 **A return Poincaré map for the Rössler flow.** *(continuation of exercise 3.1) That Poincaré return maps of figure 3.3 appear multimodal and non-invertible is an artifact of projections of a 2-dimensional return map $(R_n, z_n) \to (R_{n+1}, z_{n+1})$ onto a 1-dimensional subspace $R_n \to R_{n+1}$.*

Construct a genuine $s_{n+1} = f(s_n)$ return map by parametrazing points on a Poincaré section of the attractor figure 3.2 by a Euclidean length s computed curvilinearly along the attractor section.

This is best done (using methods to be developed in what follows) by a continuation of the unstable manifold of the 1-cycle embedded in the strange attractor, figure 12.1 (b).

(Predrag Cvitanović)

Exercise 3.3 **Arbitrary Poincaré sections.** *We will generalize the construction of Poincaré sections so that they can have any shape, as specified by the equation $U(x) = 0$.*

(a) *Start by modifying your integrator so that you can change the coordinates once you get near the Poincaré section. You can do this easily by writing the equations as*

$$\frac{dx_k}{ds} = \kappa f_k \,, \tag{3.19}$$

with $dt/ds = \kappa$, and choosing κ to be 1 or $1/f_1$. This allows one to switch between t and x_1 as the integration 'time.'

(b) *Introduce an extra dimension x_{n+1} into your system and set*

$$x_{n+1} = U(x) \,. \tag{3.20}$$

How can this be used to find a Poincaré section?

Exercise 3.4 **Classical collinear helium dynamics.** *(continuation of exercise 2.10)*

Make a Poincaré surface of section by plotting (r_1, p_1) whenever $r_2 = 0$: Note that for $r_2 = 0$, p_2 is already determined by (6.6). Compare your results with figure 35.3 (b).

(Gregor Tanner, Per Rosenqvist)

Exercise 3.5 **Hénon map fixed points.** *Show that the two fixed points (x_0, x_0), (x_1, x_1) of the Hénon map (3.15) are given by*

$$\begin{aligned} x_0 &= \frac{-(1-b) - \sqrt{(1-b)^2 + 4a}}{2a} \,, \\ x_1 &= \frac{-(1-b) + \sqrt{(1-b)^2 + 4a}}{2a} \,. \end{aligned} \tag{3.21}$$

Exercise 3.6 **How strange is the Hénon attractor?**

(a) *Iterate numerically some 100,000 times or so the Hénon map*

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 - ax^2 + y \\ bx \end{bmatrix}$$

for $a = 1.4$, $b = 0.3$. Would you describe the result as a 'strange attractor'? Why?

(b) *Now check how robust the Hénon attractor is by iterating a slightly different Hénon map, with $a = 1.39945219$, $b = 0.3$. Keep at it until the 'strange' attractor vanishes like the smile of the Chesire cat. What replaces it? Would you describe the result as a 'strange attractor'? Do you still have confidence in your own claim for the part (a) of this exercise?*

Exercise 3.7 **Fixed points of maps.** *A continuous function F is a contraction of the unit interval if it maps the interval inside itself.*

(a) *Use the continuity of F to show that a one-dimensional contraction F of the interval $[0, 1]$ has at least one fixed point.*

(b) *In a uniform (hyperbolic) contraction the slope of F is always smaller than one, $|F'| < 1$. Is the composition of uniform contractions a contraction? Is it uniform?*