Exercises

Exercise 29.1 **Dirac delta function, Lorentzian representation.** *Derive the representation (29.19)*

$$\delta(E-E_n) = -\lim_{\epsilon\to+0}\frac{1}{\pi}\mathrm{Im}\,\frac{1}{\mathrm{E}-\mathrm{E_n}+\mathrm{i}\epsilon}$$

of a delta function as imaginary part of $1/x$*.*

(Hint: read up on principal parts, positive and negative frequency part of the delta function, the Cauchy theorem in a good quantum mechanics textbook).

Exercise 29.2 **Green's function.** *Verify Green's function Laplace transform (29.16),*

$$\begin{aligned} G(q,q',E+i\varepsilon) &= \frac{1}{i\hbar}\int_0^\infty dt\, e^{\frac{i}{\hbar}Et-\frac{\varepsilon}{\hbar}t}K(q,q',t) \\ &= \sum\frac{\phi_n(q)\phi_n^*(q')}{E-E_n+i\varepsilon} \end{aligned}$$

argue that positive ϵ *is needed (hint: read a good quantum mechanics textbook).*