Exercises

Exercise 26.1 Evaluation of cycles by minimization*. *Given a symbol sequence, you can construct a guess trajectory by taking a point on the boundary of each disk in the sequence, and connecting them by straight lines. If this were a rubber band wrapped through 3 rings, it would shrink into the physical trajectory, which minimizes the action (in this case, the length) of the trajectory.*

Write a program to find the periodic orbits for your billiard simulator. Use the least action principle to extremize the length of the periodic orbit, and reproduce the periods and stabilities of 3-disk cycles, table 26.3. After that check the accuracy of the computed orbits by iterating them forward with your simulator. What is $|f^{T_p}(x) - x|$*?*

Exercise 26.2 Tracking cycles adiabatically*. *Once a cycle has been found, orbits for different system parameters values may be obtained by varying slowly (adiabatically) the parameters, and using the old orbit points as starting guesses in the Newton method. Try this method out on the 3-disk system. It works well for* $R : a$ *sufficiently large. For smaller values, some orbits change rather quickly and require very small step sizes. In addition, for ratios below* $R : a = 2.04821419\ldots$ *families of cycles are pruned, i.e. some of the minimal length trajectories are blocked by intervening disks.*

Exercise 26.3 Find cycles of the Hénon map. *Apply the method of sect. 26.1 to the Hénon map at the Hénon's parameters choice* $a = 1.4$, $b = 0.3$, *and compute all prime cycles for at least* $n \leq 6$. *Estimate the topological entropy, either from the definition (13.1), or as the zero of a truncated topological zeta function (13.21). Do your cycles agree with the cycles listed in table 26.1?*