Exercises

Exercise 16.1 $t \to 0_+$ **regularization of eigenvalue sums******.** *In taking the Laplace transform (16.23) we have ignored the $t \to 0_+$ divergence, as we do not know how to regularize the delta function kernel in this limit. In the quantum (or heat kernel) case this limit gives rise to the Weyl or Thomas-Fermi mean eigenvalue spacing (see sect. 32.1.1). Regularize the divergent sum in (16.23) following (for example) the prescription of appendix J.6 and assign to such volume term some interesting role in the theory of classical resonance spectra. E-mail the solution to the authors.*

Exercise 16.2 **General weights.** *(easy) Let f^t be a flow and $\mathcal{L}^t$ the operator*

$$\mathcal{L}^t g(x) = \int dy\, \delta(x - f^t(y)) w(t, y) g(y)$$

where w is a weight function. In this problem we will try and determine some of the properties w must satisfy.

(a) *Compute $\mathcal{L}^s \mathcal{L}^t g(x)$ to show that*

$$w(s, f^t(x)) w(t, x) = w(t + s, x)\,.$$

(b) *Restrict t and s to be integers and show that the most general form of w is*

$$w(n, x) = g(x) g(f(x)) g(f^2(x)) \cdots g(f^{n-1}(x))\,,$$

for some g that can be multiplied. Could g be a function from $\mathbb{R}^{n_1} \mapsto \mathbb{R}^{n_2}$? ($n_i \in \mathbb{N}$.)