Exercises

Exercise 30.1 WKB ansatz. *Try to show that no other ansatz other than (31.1) gives a meaningful definition of the momentum in the $\hbar \to 0$ limit.*

Exercise 30.2 Fresnel integral. *Derive the Fresnel integral*

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dx\, e^{-\frac{x^2}{2ia}} = \sqrt{ia} = |a|^{1/2} e^{i\frac{\pi}{4}\frac{a}{|a|}} \,.$$

Exercise 30.3 Sterling formula for $n!$. *Compute an approximate value of $n!$ for large n using the stationary phase approximation. Hint: $n! = \int_0^\infty dt\, t^n e^{-t}$.*

Exercise 30.4 Airy function for large arguments. *Important contributions as stationary phase points may arise from extremal points where the first non-zero term in a Taylor expansion of the phase is of third or higher order. Such situations occur, for example, at bifurcation points or in diffraction effects, (such as waves near sharp corners, waves creeping around obstacles, etc.). In such calculations, one meets Airy functions integrals of the form*

$$Ai(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dy\, e^{i(xy - \frac{y^3}{3})} \,. \tag{30.28}$$

Calculate the Airy function $Ai(x)$ using the stationary phase approximation. What happens when considering the limit $x \to 0$. Estimate for which value of x the stationary phase approximation breaks down.