H.5 Hopf's last hope

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Continue investigation initiated in
F. Christiansen, P. Cvitanović and V. Putkaradze, *"Hopf's last hope: spatiotemporal chaos in terms of unstable recurrent patterns"*, *Nonlinearity* , to appear (1996);
available on `http://www.nbi.dk/~predrag/preprints.html`

H.6 Hydrogen in external magnetic field - disociation rate

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H.7 Product of random matrices

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1. Assume you have two positive matrices A and B and that you form products of these matrices at random. How does the trace of the product grow as the number of matices increases?
2. Think of the matrices as the Jacobian matrices of some dynamical system and use that analogy to derive an expression for the asymptotic growth-rate of the trace (this is the Lyapunov exponent for a product of random matrices).
3. Write a Monte-Carlo simulator for these products.
4. Compare your results with the micro-canonical formulas of ref. [H.2].
5. Determine the Hamiltonian for an Ising model with random coupling among the spins. Assume that the coupling can only assume a finite number of values.
6. Derive the transfer matrices for the model.
7. Suppose the free-energy is averaged over the possible couplings (quenched disorder). Show that average free energy can be computed from the Lyapunov exponent for the product of random matrices.

8. Compute the heat capacity for an Ising model on a strip of width $L \sim 10$.
9. (Not in the literature) Derive an expression for the Lyapunov exponent when the distribution of matrices is continuous.

References

[H.1] F. J. Dyson. The dynamics of a disordered linear chain. *Physical Review*, 92(6):1331–1338, 1953.

[H.2] J. M. Deutsch and G. Paladin. Product of random matrices in a microcanonical ensemble. *Physical Review Letters*, 62:695–699, 1989.

[H.3] Ronnie Mainieri, "Zeta function for the Lyapunov exponent of a product of random matrices", *Physical Review Letters* **68**:1965–1968, 1992.

[H.4] Ronnie Mainieri, Cycle expansion for the Lyapunov exponent of a product of random matrices, *Chaos* **2**:91, 1992.

Other possible term projects (afløsningsopgaver)

1. Lagrangian flow, passive advection
2. thermostat (stat. mech from deterministic chaos) (from L. Rondoni??)
3. 2-colliding disks gas
4. conductivity
5. triangular Lorentz gas
6. 4-disk repeller
7. ?? (open for suggestions)